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# Mortgage choice, house price externalities, and the default rate

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ABSTRACT

We study the pathways by which borrowers and lenders influence house prices and default rates via their choices and offerings of fixed-rate and adjustable-rate mortgage products (FRMs and ARMs) in a two-period setting. We extend previous literature on mortgage choice as a tool for borrower risk screening under asymmetric information by incorporating house price externalities. The novelty in our setup is that house prices in the second period are negatively affected by the first-period default rate. We show that when these negative externalities are large, lenders may benefit by offering a lower ARM rate. This outcome, in turn, influences the likelihood of a separating equilibrium in which high-risk (low-risk) borrowers choose ARMs (FRMs) relative to a pooling equilibrium in which both high-risk and low-risk borrowers receive the same contract. When the impact of the negative house price externalities is small, it is more likely that lenders will offer pooling contracts; however, when the impact of the house price externalities is large, it is more likely that lenders will offer separating contracts. We also compare the equilibrium default rates across different contract offerings and find that when the negative house price externalities are large, the pooling FRM contract or the separating contract tends to offer the lowest default rate; however, when the negative house price externalities are small, the pooling ARM contract may result in the lowest default rate.



## 1. Introduction

Since the U.S. housing market began to decline in the second quarter of 2006, the default rate for adjustable-rate mortgage products (ARMs) has consistently exceeded that for fixed-rate mortgages (FRMs),[1] and this difference can be attributed to at least two factors. First, some theory suggests that ARMs may be preferred among high-risk borrowers (Posey and Yavas, 2001). The fact that ARMs tend to be more popular among subprime borrowers is consistent with this suggestion (Pennington-Cross and Ho, 2010). Second, ARMs may also increase default risk via a payment shock if interest rates increase sufficiently to leave the borrower liquidity constrained, given his income. For these reasons, the popularity of ARMs prior to 2005 has been implicated as a factor contributing to the recent subprime mortgage crisis (Scanlon et al., 2008; Pennington-Cross and Ho, 2010).

Beyond the direct contribution of ARMs to the default rate, some researchers have also suggested that geographic proximity to alternative mortgage products, such as hybrid ARMs, may create spillovers that lead to higher default among nearby property owners (Agarwal et al., 2012), and thus potentially among borrowers with other types of mortgage products. Although general economic conditions,

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[1] For default rates by mortgage type, see the Mortgage Banker's Association's National Mortgage Delinquency Survey.

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such as high unemployment rates, have likely also contributed to recent increases in default rates across mortgage products,[2] the fact that the default rate on FRMs has also increased in recent years appears consistent with the idea that house price externalities may exist. Moreover, a variety of empirical evidence suggests that house price declines and equity-driven defaults are contagious, for reasons of both house prices and social norms/networks (Immergluck and Smith, 2006; Schuetz et al., 2008; Harding et al., 2009; Lin et al., 2009; Goodstein et al., 2011; Guiso et al., 2011; Campbell et al., 2011), and that negative equity plays a larger role than unemployment rates in driving defaults (Goodman et al., 2010). Thus, there is an implicit conduit running from the distribution of loan products to the house price level, and from house prices to default rates.[3]

However, the contribution of loan product to default risk has also been found to vary with the economic environment. An FRM can protect a borrower from inflation but can raise equity-driven default risk. ARMs may exhibit a lower or higher default rate than FRMs, depending both on the movement of interest rates and on whether principal payments are deferred, as in the case of interest-only or option ARMs (Vandell, 1978; LaCour-Little and Yang, 2010; Campbell and Cocco, 2011). Thus, the menu of mortgage contracts that minimizes the equilibrium default rate under house price externalities is not obvious *a priori*.

In this paper, we study the pathways by which borrowers and lenders influence house prices and default rates via their choices and offerings of FRM and ARM products.[4] We extend the model of Posey and Yavas (2001), who demonstrate that mortgage product can be used as a risk screening tool, to incorporate house price externalities. In the model of Posey and Yavas (2001), borrowers are either high risk or low risk according to their likelihood of having a negative income shock. As long as the disutility of default is sufficiently high, high-risk borrowers tend to prefer ARM contracts, while low-risk borrowers tend to prefer FRM contracts. Intuitively, for an ARM contract, the potential costs associated with a rise in interest rates are outweighed by the potential benefits of a decline in interest rates for high risks but not low risks. Thus, high risks will experience a lower expected default rate under an ARM than under an FRM. Posey and Yavas (2001) find that two separating equilibria and two pooling equilibria exist in under asymmetric information. One of the separating equilibria provides positive profits to the lender, while the other does not. In both cases, high risks receive the ARM and low risks receive the FRM. These separating equilibria become increasingly likely relative to pooling contracts as the proportion of high risks in the population increases, and as the difference in the likelihoods of an income shock becomes greater.

In our model, borrowers similarly self-select into either FRMs or ARMs according to risk type. However, after defaults from income shocks are observed, house prices are determined, and an additional wave of defaults can occur based on the change in house prices. We find that house price externalities likewise have an impact on the likelihood of separating versus pooling equilibria, and on the equilibrium ARM interest rate, because they mediate the equilibrium default rate under different mortgage product menus via their interplay with the ARM interest rate.

A variety of related papers have considered mortgage choice under asymmetric information; beyond the work of Posey and Yavas (2001), those most similar to ours include papers by Brueckner (2000), Ben-Sharar (2006), and LaCour-Little and Yang (2010). To the best our knowledge, however, our paper is the first to explicitly consider the linkages among mortgage product choices, house price contagion, and equilibrium default rates under asymmetric information. Our paper complements existing borrower screening models by incorporating feedback from house prices to default rates via an additional period in which the price level is permitted to adjust and new defaults to occur. We also derive implications for the aggregate default rate, which may now depend on house price externalities, and provide insight into how to control the default rate in different situations.

In the following section, we present the model. In subsequent sections, we derive and interpret the results under both full information, in which the borrower type is public information, and under asymmetric information, in which the borrower's type is unknown to the lenders. In the final section, we provide conclusions.

## 2. Model

The model closely follows that of Posey and Yavas (2001). Consider a competitive lending market where lenders offer fixed-rate mortgages (FRMs) and/or adjustable-rate mortgages (ARMs). We consider interest-only loans with the loan amount as a balloon payment at the end of the term. The loan amount is normalized to \$1. There are two periods. The lenders borrow short-term (one year) at the spot market rate. At time $t = 0$, each borrower has a current income of $Y$, at which he qualifies for either an ARM or an FRM loan. However, the borrower's income may change in future periods, $t = 1, 2$. There are two types of borrowers: type H (high risk) borrowers have a higher probability than type L (low risk) borrowers of facing a reduction in income, which may fall to $y < Y$ at $t = 1$. Income remains constant from period 1 to period 2. The probability that a type $j$ borrower experiences a decline in income is given by $p_j, j = L, H, p_H > p_L$. Let $\lambda_j$ be the

[2] For example, Makarov and Plantin (2009) show that correlated income shocks can cause systemic defaults.

[3] More generous loan terms may also inflate house purchase prices by relaxing credit constraints and thus influence default rates. In this paper, we focus on the way in which mortgage choice relates to post-purchase house price movements and default rates, given an exogenous purchase price; however, we hope to consider the role of endogenous purchase prices in future work.

[4] While our analysis is partly motivated by recent housing market events and the empirical research mentioned above, please note that our goal here is not to model recent events in realistic detail or to provide concrete policy recommendations about how to avoid similar events in the future. Rather, we aim (more generally) to investigate and illustrate the dynamics of borrower and lender interaction in the context of both borrower income shocks and negative house price externalities, and to derive intuition about how the menu of mortgage contracts is related to the overall default rate in this context.

proportion of type $j$ borrowers in the population, such that $\sum_{j=L,H}\lambda_j = 1$. We assume that lenders are risk neutral and that the borrowers are risk averse. The discount factor is $\delta$.

Let $r+\varepsilon_t$ be the interest rate at which a lender can borrow at time $t = 1, 2$, where $\varepsilon_t$ is a random variable with density $f_t(\varepsilon_t)$ and cumulative density $F_t(\varepsilon_t)$ on $[\underline{\varepsilon}_t, \overline{\varepsilon}_t]$. At the beginning of the first period ($t = 0$), each lender decides which contract(s) to offer, FRM and/or ARM, and sets the interest rate for each type of mortgage contract. For an FRM contract, the interest rate is denoted by $i$. For an ARM contract, the interest rate is given by $r+\alpha+\varepsilon_t$, where $\alpha$ represents the lender's margin to compensate for potential default at time $t = 1, 2$.

The borrower chooses a contract and buys a house at unit price $p_0 = 1$ at the beginning of the first period ($t = 0$). Since the loan is an interest-only product, the borrower pays interest only at the end of the first period ($t = 1$) and pays both interest and principle at the end of second period ($t = 2$). The borrower can default in either period. If he defaults at the end of the first period, the house price at the end of the second period is negatively impacted and is represented by $p_2 = 1 - qm$, where $q < 1$ is the house value depreciation coefficient and $m$ is the rate of default at the end of the first period. We assume that the borrower can only sell the house in the second period and that $y < i < Y$. Thus, an FRM borrower defaults at the end of the first period if his income falls. In contrast, an ARM borrower may or may not default if his income falls, because his payment at the end of the first period is $r+\alpha+\varepsilon_1$, which could be greater or less than $Y$ ($y$). Consequently, an FRM borrower will only default in the second period if the house value falls sufficiently, while whether an ARM borrower defaults depends on both the house value and the second-period payment. If the price of the house is too low in the second period, the borrower will choose to default. An FRM borrower $j$ will default at the end of the second period if the sum of the house value and income, $p_2(m) + Y$, is less than the payment, $1 + i_j$.[5] Since a borrower $j$ with a reduction in income may not default at $t = 2$ under an ARM contract, default occurs if the sum of the house value, income and the remainder of the first period income, $p_2(m) + Y$ or $p_2(m) + y$, is less than the payment, $1 + r + \alpha_j + \varepsilon_2$. For now, assume that $r+\alpha+\underline{\varepsilon}_t < y < i < Y < r+\alpha+\overline{\varepsilon}_t$. Finally, the borrower sells the house at time $t = 2$.

## 3. Borrower's problem

Each borrower chooses a mortgage contract to maximize his expected utility. We assume that each borrower has an identical utility function that is strictly concave in wealth. When a default occurs, each borrower experiences a disutility $D$ that includes all the costs associated with a default, such as a lowered credit rating, emotional and physical distress, and so forth. Following the setup by Posey and Yavas (2001), the expected utility of a borrower of type $j$, $j = H, L$, from an FRM contract is

$$V^F(i,p_j,m) = (1-p_j)U(Y-i) + p_j(U(0)-D) + \delta(1-p_j) \times \left(U(Max[Y-i-qm,0]) - D\left(1 - \frac{Max[Y-i-qm,0]}{Y-i-qm}\right)\right) \quad (1)$$

where the first two terms represent borrower $j$'s first-period utility and the third term represents borrower $j's$ second-period utility. In the first period, borrower $j$ pays the interest $i$ when his income remains $Y$; thus, the borrower's remaining wealth is $Y - i$. When his income falls to $y$, borrower $j$ instead pays $y$ to the lender, and his remaining wealth is zero. In the second period, the house value falls below 1 due to defaults in the first period. If the sum of equity and income is greater than the total payment (including both interest and principle), the borrower will not default, and his remaining wealth is $Y - i + p_2 - 1$, which is $Y - i - qm$. However, if the sum of equity and income together is less than the total payment, the borrower will default, and his remaining wealth is zero. The expression $1 - \frac{Max[Y-i-qm,0]}{Y-i-qm}$ is simply equal to 1 when $Y - i - qm \leqslant 0$ and is equal to zero when $Y - i - qm > 0$. Assume that $Y$ is large enough so that $Y - i - qm > 0$. We then have

$$V^F(i,p_j,m) = (1-p_j)U(Y-i) + p_j(U(0)-D) + \delta(1-p_j)U(Y-i-qm) \quad (2)$$

If borrower $j$ chooses an ARM contract, his expected utility is given by

$$V^A(\alpha,p_j,m) = (1-p_j)\left\{\int_{\underline{\varepsilon}_1}^{Y-r-\alpha}(U(Y-r-\alpha-\varepsilon_1)+\delta\widehat{A})f_1(\varepsilon_1)d\varepsilon_1 + \int_{Y-r-\alpha}^{\overline{\varepsilon}_1}(U(0)-D)f_1(\varepsilon_1)d\varepsilon_1\right\} + p_j\left\{\int_{\underline{\varepsilon}_1}^{y-r-\alpha}(U(y-r-\alpha-\varepsilon_1)+\delta\widehat{B})f_1(\varepsilon_1)d\varepsilon_1 + \int_{y-r-\alpha}^{\overline{\varepsilon}_1}(U(0)-D)f_1(\varepsilon_1)d\varepsilon_1\right\}.$$

where

$$\widehat{A} = \int_{\underline{\varepsilon}_2}^{Min[\overline{\varepsilon}_2,Y-r-\alpha-qm]} U(Y-r-\alpha-\varepsilon_2-qm)f_2(\varepsilon_2)d\varepsilon_2 + \int_{Max[\underline{\varepsilon}_2,Y-r-\alpha-qm]}^{\overline{\varepsilon}_2}(U(0)-D)f_2(\varepsilon_2)d\varepsilon_2 \text{ and}$$

$$\widehat{B} = \int_{\underline{\varepsilon}_2}^{Min[\overline{\varepsilon}_2,y-r-\alpha-qm]} U(y-r-\alpha-\varepsilon_2-qm)f_2(\varepsilon_2)d\varepsilon_2 + \int_{Max[\underline{\varepsilon}_2,y-r-\alpha-qm]}^{\overline{\varepsilon}_2}(U(0)-D))f_2(\varepsilon_2)d\varepsilon_2$$

The expression $\widehat{A}$ represents utility at time $t = 2$ when there is no reduction in income, while the expression $\widehat{B}$ represents utility in the second period when income falls. It is easy to see that $\widehat{A} > \widehat{B}$. The first term represents borrower $j$'s utility when there is no reduction in income, while the second term reflects borrower $j's$ utility when

[5] In this setup, the borrowers choose to default strategically and may default even if income is greater than the payment. Moreover, as we discuss in more detail below, we consider the default decision in the context of a sufficiently high disutility of default.

income falls. In addition, borrower $j$ pays interest $r+\alpha+\varepsilon_1$ at the end of the first period if there is no default; otherwise, he will choose to default, which leads to a remaining wealth of zero and to disutilities of default. When he does not default at the end of the first period, borrower $j$ will proceed to the second period. At that point, his action depends on the realized house value $p_2(m)$ and the random variable $\varepsilon_2$. The higher $p_2(m)$ is, the less likely it is that an equity-driven default will occur. The borrower defaults if the house value is so low or $\varepsilon_2$ is so high that the sum of equity and income is less than the total payment, $r+\alpha+\varepsilon_2+1$.

Suppose that both FRM and ARM contracts are offered and the contracts are designed so that the low-risk borrowers are attracted to the FRM contract and the high-risk borrowers are attracted to the ARM contract, and let $\Delta V(i,\alpha,p_L) = V^F(i,p_L,m) - V^A(\alpha,p_L,\widetilde{m})$ be the difference in the utilities derived from an FRM contract and an ARM contract for low-risk borrowers. The reason why the first-period default rates are different in $V^F$ and $V^A$ is as follows: If the low-risk borrowers were to choose the ARM contract instead of the FRM contract, given that the high-risk borrowers choose the ARM contract as well, the expression for the first-period default rate would be

$$\widetilde{m}_L = \sum_{j=L,H}\lambda_j\left\{(1-p_j)\int_{Y-r-\alpha}^{\bar{\varepsilon}_1} f_1(\varepsilon_1)d\varepsilon_1 + p_j\int_{y-r-\alpha}^{\bar{\varepsilon}_1} f_1(\varepsilon_1)d\varepsilon_1\right\}$$

instead of $V^F(i,p_j,m)$ where

$$m = \lambda_H\left\{(1-p_H)\int_{Y-r-\alpha}^{\bar{\varepsilon}_1} f(\varepsilon_1)d\varepsilon_1 + p_H\int_{y-r-\alpha}^{\bar{\varepsilon}_1} f(\varepsilon_1)d\varepsilon_1\right\} + \lambda_L p_L.$$

Conversely, if the high-risk borrowers were to choose the FRM contract instead, given that the low-risk borrowers choose the FRM contract as well, the expression for the first-period default rate would be

$$\widetilde{m}_H = \sum_{j=L,H}\lambda_j p_j$$

instead of $m$. Thus, we have $\Delta V(i,\alpha,p_H) = V^F(i,p_H,\widetilde{m}_H) - V^A(\alpha,p_H,m)$.

It can be shown easily that $\frac{\partial \Delta V(i,\alpha;p_j)}{\partial i} < 0$ and $\frac{\partial \Delta V(i,\alpha;p_j)}{\partial \alpha} > 0$. Let $i(\alpha,p,j)$ be the FRM rate such that $\Delta V(i,\alpha;p_j) = 0$. Then, we have the following results:

**Lemma 1.** *There exists a sufficiently large D such that* $\frac{\partial \Delta V(i,\alpha;p_j)}{\partial p_j} < 0$.

**Proposition 2.** *If D is sufficiently large such that* $\frac{\partial \Delta V(i,\alpha;p_j)}{\partial p_j} < 0$, *then* $\frac{\partial i(\alpha;p_j,j)}{\partial p_j} < 0$.

Similar to the result of Posey and Yavas (2001), Proposition 2 shows that, as $p_j$ increases, the difference between the utility from choosing an FRM contract and the utility from choosing an ARM contract becomes smaller. Since $p_H > p_L$, low-risk borrowers will not prefer an ARM contract, while high-risk borrowers prefer an FRM contract. For the rest of the paper, we assume that $D$ is sufficiently large.

## 4. Lender's problem: full information

We first consider the scenario in which the type of borrower is known to the lender. Assuming a competitive market, we develop the zero-profit contracts for each type of borrower.

### 4.1. FRM contracts

The lender's expected payoff when making an FRM loan to a type $j$ borrower is characterized by

$$\Pi^F(i_j,p_j) = (1-p_j)(i_j - r - E(\varepsilon_1)) + p_j(y - r - E(\varepsilon_1)) + \delta(1-p_j)(i_j - r - E(\varepsilon_2))$$

since $Y - qm - i_j \geqslant 0$. The lender's earnings expression at the end of the first period includes two parts. If the borrower's income remains $Y$ at the end of the first period, he does not default, and the lender earns $i_j - r - E(\varepsilon_1)$; if the borrower's income decreases to $y$, he defaults, and the lender only collects $y$. Note that the house value at $t=1$ remains the same as at the beginning of the first period, at $t=0$. At the end of the second period, the payments are collected from the borrowers with income $Y$, and these payments are discounted by $\delta$. Unlike in the first period, the lender's earnings expression at $t=2$ includes only one term. Since $Y - qm - i_j \geqslant 0$ by assumption, the expected earnings are $i_j - r - E(\varepsilon_1)$. Differentiating $\Pi^F(i_j,p_j)$ with respect to $i_j$ and $p_j$, we have the following lemma.

**Lemma 3.** $\frac{d\Pi^F}{di_j} > 0$ *and* $\frac{d\Pi^F}{dp_j} < 0$.

### 4.2. ARM contracts

Now consider an ARM contract. The lender's expected payoff for making an ARM loan to a type $j$ borrower is characterized by

$$\begin{aligned}\Pi^A(\alpha_j,p_j,m) = (1-p_j)&\left\{\int_{\underline{\varepsilon}_1}^{Y-r-\alpha_j}(r+\alpha_j+\varepsilon_1+\delta A)f_1(\varepsilon_1)d\varepsilon_1\right.\\&\left.+\int_{Y-r-\alpha_j}^{\bar{\varepsilon}_1} Yf_1(\varepsilon_1)d\varepsilon_1\right\} + p_j\left\{\int_{\underline{\varepsilon}_1}^{y-r-\alpha_j}(r+\alpha_j\right.\\&\left.+\varepsilon_1+\delta B)f_1(\varepsilon_1)d\varepsilon_1 + \int_{y-r-\alpha_j}^{\bar{\varepsilon}_1} yf_1(\varepsilon_1)d\varepsilon_1\right\}\\&-\int_{\underline{\varepsilon}_1}^{\bar{\varepsilon}_1}(r+\varepsilon_1)f_1(\varepsilon_1)d\varepsilon_1\\&-\delta\int_{\underline{\varepsilon}_2}^{\bar{\varepsilon}_2}(r+\varepsilon_2)f_2(\varepsilon_2)d\varepsilon_2 \qquad (3)\end{aligned}$$

and

$$\begin{aligned}A &= \int_{\underline{\varepsilon}_2}^{Min[\bar{\varepsilon}_2,Y-r-\alpha_j-qm]}(r+\alpha_j+\varepsilon_2)f_2(\varepsilon_2)d\varepsilon_2\\&\quad+\int_{Max[\underline{\varepsilon}_2,Y-r-\alpha_j-qm]}^{\bar{\varepsilon}_2}(Y-qm)f_2(\varepsilon_2)d\varepsilon_2\\B &= \int_{\underline{\varepsilon}_2}^{Min[\bar{\varepsilon}_2,y-r-\alpha_j-qm]}(r+\alpha_j+\varepsilon_2)f_2(\varepsilon_2)d\varepsilon_2\\&\quad+\int_{Max[\underline{\varepsilon}_2,y-r-\alpha_j-qm]}^{\bar{\varepsilon}_2}(y-qm)f_2(\varepsilon_2)d\varepsilon_2\end{aligned}$$

The expression $A$ represents the lender's earnings at time $t = 2$ when there is no reduction in income, while the expression $B$ represents the lender's earnings in the second period when the borrower's income falls. Under an ARM contract, a borrower with reduced income may not default in the first period if $\varepsilon_1$ is low enough. With probability $1 - p_j$, borrower $j$ will not experience a reduction in income; thus, the first term in expression (3) represents total expected earnings when there is no reduction in income, while the second term represents total expected earnings when there is a reduction in income. At time $t = 2$, depending on the house value $p_2$ and the random variable $\varepsilon_2$, the borrower will choose to default if the sum of equity and income , $p_2(m) + Y$ or $p_2(m) + y$, is less than the payment $1 + r + \alpha_j + \varepsilon_2$. Differentiating $\Pi^A(\alpha_j, p_j, m)$ with respect to $\alpha_j, p_j$ and $m$ yields the following results:

**Lemma 4.** *$\frac{d\Pi^A}{dp_j} < 0, \frac{\partial \Pi^A}{\partial m} < 0$, and $\frac{d\Pi^A}{d\alpha_j} \lesseqgtr 0$.*

**Proof.** It is a straightforward matter to prove $\frac{d\Pi^A}{dp_j} < 0$ and $\frac{\partial \Pi^A}{\partial m} < 0$; therefore, we omit the proof. Taking the derivatives of $A$ and $B$ with respect to $\alpha_j$, we obtain

$$\frac{dA}{d\alpha_j} = \int_{\underline{\varepsilon}_2}^{Min[\bar{\varepsilon}_2, Y-r-\alpha_j-qm]} f_2(\varepsilon_2) d\varepsilon_2 - q\frac{dm}{d\alpha_j} \times \int_{Max[\underline{\varepsilon}_2, Y-r-\alpha_j-qm]}^{\bar{\varepsilon}_2} f_2(\varepsilon_2) d\varepsilon_2 \lesseqgtr 0$$

and

$$\frac{dB}{d\alpha_j} = \int_{\underline{\varepsilon}_2}^{Min[\bar{\varepsilon}_2, y-r-\alpha_j-qm]} f_2(\varepsilon_2) d\varepsilon_2 - q\frac{dm}{d\alpha_j} \times \int_{Max[\underline{\varepsilon}_2, y-r-\alpha_j-qm]}^{\bar{\varepsilon}_2} f_2(\varepsilon_2) d\varepsilon_2 \lesseqgtr 0.$$

These results show that the lender's earnings in the second period may or may not increase as the ARM rate increases, depending on the magnitude of the negative house price externalities. In the presence of large negative house price externalities, a higher ARM rate triggers an additional loss of earnings that would not otherwise occur. That is, $q\frac{dm}{d\alpha_j}\int_{Max[\underline{\varepsilon}_2, Y-r-\alpha_j-qm]}^{\bar{\varepsilon}_2} f_2(\varepsilon_2)d\varepsilon_2$ under high income, or $q\frac{dm}{d\alpha_j}\int_{Max[\underline{\varepsilon}_2, y-r-\alpha_j-qm]}^{\bar{\varepsilon}_2} f_2(\varepsilon_2)d\varepsilon_2$ under low income. This loss, in turn, affects the change in total earnings, which is represented by

$$\frac{d\Pi^A}{d\alpha_j} = (1-p_j)\left\{-\delta A f_1(Y-r-\alpha_j) + \left(1 + \delta\frac{dA}{d\alpha_j}\right)F_1(Y-r-\alpha_j)\right\} + p_j\left\{-\delta B f_1(y-r-\alpha_j) + \left(1+\delta\frac{dB}{d\alpha_j}\right)F_1(y-r-\alpha_j)\right\}.$$

It can be seen that the sign of expression (4) remains undetermined. □

To understand $\frac{d\Pi^A}{d\alpha_j}$ further, we analyze the change in first-period earnings as well. Let $\Pi_i^A$ denote the earnings in period $i$; then,

$$\frac{d\Pi_1^A}{d\alpha_j} = (1-p_j)F_1(Y-r-\alpha_j) + p_j F_1(y-r-\alpha_j) > 0$$

This expression indicates that the lender's first-period earnings are always increasing in the ARM rate. We then have the following proposition:

**Proposition 5.** *For a sufficiently small $\delta, \frac{d\Pi^A}{d\alpha_j} > 0$.*

When the discount factor is sufficiently small, second-period earnings become insignificant; thus, the negative effect of decreasing housing prices does not cause a major shift in total earnings. As a result, the effect on first-period earnings dominates the change in total earnings. Since $\frac{d\Pi_1^A}{d\alpha_j} > 0$, total earnings will increase as well.

Assume that $D$ is sufficiently large, by Proposition 2, so that the lender never offers an FRM contract to high-risk borrowers while offering an ARM contract to low-risk borrowers. We study the following three cases:

- Case I: The lender offers FRM contracts only, with rate $i_L$ offered to low-risk borrowers and rate $i_H$ offered to high-risk borrowers.
- Case II: The lender offers ARM contracts only, with rate $\alpha_L$ offered to low-risk borrowers and rate $\alpha_H$ offered to high-risk borrowers.
- Case III: The lender offers an FRM contract with rate $i_L$ to low-risk borrowers and an ARM contract with rate $\alpha_H$ to high-risk borrowers.

One can see easily that the equilibrium FRM rates and ARM rates are case dependent. Thus, we denote by $i_j^k$ and $\alpha_j^k$ the equilibrium FRM rate and ARM rate for borrower $j$ in case $k$. Moreover, let $m^k$ and $T^k$ be the first-period and total default rates in case $k$. In the following sections, we derive the equilibrium conditions and the default rate for each case.

*Case I: the lender offers FRM contracts to both borrower types.*

In this case, the FRM rates $i_L^I$ and $i_H^I$ solve $\Pi^F(i_j^I, p_j, m^I) = 0$ subject to $V^F(i_J^I, p_j, m^I) \geqslant 0$. We have $m^I = \sum_{j=L,H}\lambda_j p_j$ and $T^I = \sum_{j=L,H}\lambda_j p_j$.

*Case II: the lender offers ARM contracts to both borrower types.*

In this case, the ARM rates $\alpha_j^{II}$ solve

$$\begin{cases} \Pi^A(\alpha_H^{II}, p_H, m^{II}) = 0 \\ \Pi^A(\alpha_L^{II}, p_L, m^{II}) = 0 \end{cases}$$

subject to $V^A(\alpha_j^{II}, p_j, m^{II}) \geqslant 0$. We have

$$m^{II}(\alpha_L^{II}, \alpha_H^{II}) = \sum_{j=L,H}\lambda_j\left\{(1-p_j)\int_{Y-r-\alpha_j^{II}}^{\bar{\varepsilon}_1} f_1(\varepsilon_1)d\varepsilon_1 + p_j\int_{y-r-\alpha_j^{II}}^{\bar{\varepsilon}_1} f_1(\varepsilon_1)d\varepsilon_1\right\}.$$

and

$$T^{II} = \sum_{j=L,H} \lambda_j \Bigg\{ (1-p_j) \Bigg\{ \int_{\underline{\varepsilon}_1}^{Y-r-\alpha_j^{II}} \int_{Max[\underline{\varepsilon}_2, Y-r-\alpha_j^{II}-qm^{II}]}^{\bar{\varepsilon}_2} f_2(\varepsilon_2)d\varepsilon_2 f_1(\varepsilon_1)d\varepsilon_1 + \int_{Y-r-\alpha_j^{II}}^{\bar{\varepsilon}_1} f_1(\varepsilon_1)d\varepsilon_1 \Bigg\} + p_j \Bigg\{ \int_{\underline{\varepsilon}_1}^{y-r-\alpha_j^{II}} \int_{Max[\underline{\varepsilon}_2, y-r-\alpha_j^{II}-qm]}^{\bar{\varepsilon}_2} f_2(\varepsilon_2)d\varepsilon_2 f_1(\varepsilon_1)d\varepsilon_1 + \int_{y-r-\alpha_j^{II}}^{\bar{\varepsilon}_1} f_1(\varepsilon_1)d\varepsilon_1 \Bigg\} \Bigg\}.$$

Taking the derivatives of $m^{II}$ with respect to $\alpha_j$, we obtain

$$\frac{dm^{II}}{d\alpha_j} = \sum_{j=L,H} \lambda_j \{(1-p_j) f_1(Y-r-\alpha_j) + p_j f_1(y-r-\alpha_j)\} > 0$$

*Case III: the lender offers high-risk borrowers an ARM contract and low-risk borrowers an FRM contract.*

In this case, $i_L^{III}$ and $\alpha_H^{III}$ satisfy

$$\begin{cases} \Pi^F(i_L^{III}, p_L, m^{III}) = 0 \\ \Pi^A(\alpha_H^{III}, p_H, m^{III}) = 0 \end{cases}$$

subject to $V^A(\alpha_H^{III}, p_H, m^{III}), V^F(i_L^{III}, p_L, m^{III}) \geqslant 0$. We have

$$m^{III}(\alpha_H^{III}) = \lambda_H \Bigg\{ (1-p_H) \int_{Y-r-\alpha_H^{III}}^{\bar{\varepsilon}_1} f(\varepsilon_1)d\varepsilon_1 + p_H \int_{y-r-\alpha_H^{III}}^{\bar{\varepsilon}_1} f(\varepsilon_1)d\varepsilon_1 \Bigg\} + \lambda_L p_L. \tag{5}$$

and

$$T^{III} = \lambda_L p_L + \lambda_H (1-p_H) \Bigg\{ \int_{\underline{\varepsilon}_1}^{Y-r-\alpha_{H^{III}}} \int_{Max[\underline{\varepsilon}_2, Y-r-\alpha_{H^{III}}-qm^{III}]}^{\bar{\varepsilon}_2} f_2(\varepsilon_2)d\varepsilon_2 f_1(\varepsilon_1)d\varepsilon_1 + \int_{Y-r-\alpha_{H^{III}}}^{\bar{\varepsilon}_1} f_1(\varepsilon_1)d\varepsilon_1 \Bigg\} + \lambda_H p_H \Bigg\{ \int_{\underline{\varepsilon}_1}^{y-r-\alpha_{H^{III}}} \int_{Max[\underline{\varepsilon}_2, y-r-\alpha_{H^{III}}-qm^{III}]}^{\bar{\varepsilon}_2} f_2(\varepsilon_2)d\varepsilon_2 f_1(\varepsilon_1)d\varepsilon_1 + \int_{y-r-\alpha_{H^{III}}}^{\bar{\varepsilon}_1} f_1(\varepsilon_1)d\varepsilon_1 \Bigg\}; \tag{6}$$

As in Case II, we obtain $\frac{dm^{III}}{d\alpha_j^{III}} > 0$.

### 4.3. Case comparison

**Proposition 6.** If $\frac{d\Pi^A}{d\alpha_j^k} < 0$ and $p_L$ is sufficiently small, then $\alpha_H^{II} < \alpha_H^{III}$; if $\frac{d\Pi^A}{d\alpha_j^k} < 0$ and $p_L$ is sufficiently large, then $\alpha_H^{II} > \alpha_H^{III}$.

Proposition 6 provides sufficient conditions for the ranking of ARM rates in Cases II and III. The condition $\frac{d\Pi^A}{d\alpha_j^k} < 0$ indicates that the negative house price externality has a large impact on second-period earnings and that total earnings decrease in the ARM rates; moreover, the condition that $p_L$ be sufficiently small implies that low-risk borrowers have a very low probability of a negative income shock. The comparison of ARM rates is crucial in comparing the total default rate, as we will show in the following proposition. It is obvious that $T^I > T^{III}$, so we focus on the comparison of $T^{II}$ and $T^{III}$.

**Proposition 7.** *If $p_L$ is sufficiently small and $\alpha_H^{II} > \alpha_H^{III}$, then $T^I, T^{II} > T^{III}$. If $p_L$ is sufficiently large and $\alpha_H^{II} < \alpha_H^{III}$, then $T^{II} < T^{III} < T^I$.*

Proposition 7 provides sufficient conditions for the rankings of total default rates. It suggests that the contracts in Case III incur the lowest default rate when $p_L$ is sufficiently small and $\alpha_H^{II} > \alpha_H^{III}$, while the contracts in Case II incur the lowest default rate when $p_L$ is sufficiently large and $\alpha_H^{II} < \alpha_H^{III}$. Intuitively, if $p_L$ is sufficiently small, the contract which avoids default by low-risk borrowers at the high-income level lowers the total default rate in the market. This is indeed the case if low-risk borrowers are offered FRM contracts, since by assumption low-risk borrowers do not default at the high-income level under FRM contracts. The condition $\alpha_H^{II} > \alpha_H^{III}$ induces a higher default rate for high-risk borrowers in Case II than in Case III. As a result, the total default rate is lower in Case III than in Case II. On the other hand, if $p_L$ is sufficiently large, an FRM contract increases the default rate among low-risk borrowers and should, therefore, be avoided in order to lower the total default rate. Therefore, under the condition that $\alpha_H^{II} < \alpha_H^{III}$, offering ARM contracts to both types of borrowers reduces the total default rate in the market.

This proposition also indicates that, as $p_L$ increases, the marginal increase in the total default rate becomes smaller in Case II than in Case III, so that, after a certain level, it becomes favorable to implement the contracts in Case II in order to control the total default rate. Therefore, we have the following further result:

**Proposition 8.** $\frac{\partial T^{III} - T^{II}}{\partial p_L} > 0, \frac{\partial T^{III} - T^{II}}{\partial Y} > 0$ *and* $\frac{\partial T^{III} - T^{II}}{\partial y} > 0.$

Proposition 8 shows that, as the values of $p_L$, $Y$, and $y$ increase, it is more likely that the contracts in Case II induce a lower default rate. As income increases, the probability of default goes down for borrowers under ARM contracts, while the probability of default remains the same for borrowers under FRM contracts. With regard to the effect of a change in $p_L$, the marginal decrease in the default rate in Case II is larger than the one in Case III. Consequently, the contracts in Case II become more favorable for controlling the default rate.

## 5. Lender's problem: asymmetric information

Under asymmetric information when the borrower's type is private information, lenders have to offer the same contract to all borrowers. In this case, two outcomes may arise: a *pooling equilibrium* in which lenders offer one mortgage type (either an FRM contract or an ARM contract); or, alternatively, a *separating equilibrium* in which lenders offer both FRM and ARM contracts and borrowers self-select into each contract.

### 5.1. Pooling equilibria

A pooling FRM contract $i^*$ solves

$$\lambda \Pi^F(i^*, p_L, m^I) + (1-\lambda)\Pi^F(i^*, p_H, m^I) = 0,$$

and a pooling ARM contract $\alpha^*$ solves

$$\lambda \Pi^A(\alpha^*, p_L, m^{II}(\alpha^*)) + (1-\lambda)\Pi^A(\alpha^*, p_H, m^{II}(\alpha^*)) = 0.$$

To derive the conditions for both pooling and separating equilibria, we need to compare $i^*$ with $i_L^{III}$, and $\alpha^*$ with

$\alpha_H^{III}$. Recall that $i_L^{III}$ and $\alpha_H^{III}$ are the rates offered under symmetric information. To simplify the analysis, we assume $i^* > i_L^{III}$.[6] One can see easily that if $\frac{d\Pi^A}{d\alpha_j} < 0, \alpha^* > \alpha_H^{II}$; otherwise, $\alpha^* < \alpha_H^{II}$.[7] Therefore, we have the following comparison between $\alpha^*$ and $\alpha_H^{III}$:

**Lemma 9.** *If $\frac{d\Pi^A}{d\alpha_j} < 0$ and $\alpha_H^{II} > \alpha_H^{III}$, we have $\alpha^* > \alpha_H^{III}$; if $\frac{d\Pi^A}{d\alpha_j} > 0$ and $\alpha_H^{II} < \alpha_H^{III}$, we have $\alpha^* < \alpha_H^{III}$. Otherwise, $\alpha^* \lesseqgtr \alpha_H^{III}$.*

**Proof.** We can show easily that if $\frac{d\Pi^A}{d\alpha_j} < 0, \alpha^* > \alpha_H^{II}$; otherwise, $\alpha^* < \alpha_H^{II}$. We are able to compare $\alpha^*$ with $\alpha_H^{II}$ because the lenders only offer ARM contracts in both situations, thus they have the same functional form for the first period default rate $m$, whereas, under case III with symmetric information, the lenders offer both ARM and FRM contract. □

Lemma 9 states sufficient conditions for comparing $\alpha^*$ and $\alpha_H^{III}$. It implies that $\alpha^* > \alpha_H^{III}$ is more likely to occur if $\frac{d\Pi^A}{d\alpha_j} < 0$, and that $\alpha^* < \alpha_H^{III}$ is more likely to occur if $\frac{d\Pi^A}{d\alpha_j} > 0$. When $\frac{d\Pi^A}{d\alpha_j^k} < 0$, the ARM rate offered to high-risk borrowers is smaller than the ARM rate offered to low-risk borrowers. As a result, the pooling ARM rate may be higher than the ARM rate offered to the high-risk borrowers in a separating contract. This result is surprising given the conventional wisdom that, since the lender is not able to screen the low-risk borrowers and the high-risk borrowers under a pooling contract, the pooling contract rate should always be lower than the rate offered to the high-risk borrowers.

Recall that the condition $\frac{d\Pi^A}{d\alpha_j^k} < 0$ indicates that the negative house price externality has a large impact on second-period earnings, so that total earnings decrease in ARM rates. Therefore, the implication is that negative house price externalities provide a favorable environment for $\alpha^* > \alpha_H^{III}$ to occur. This indeed plays a crucial role in determining the likelihood of a separating contract or a pooling contract, as we will show in the following propositions.

**Proposition 10.** *Suppose $\alpha^* > \alpha_H^{III}$; then there exists a pooling equilibrium with an ARM rate $\alpha^*$ if and only if 1) $i^* \geqslant i(\alpha^*, p_L, m^I, m^{II})$ and 2) $i(\alpha_H^{III}, p_H, m^I, m^{III}) < i(\alpha_H^{III}, p_L, m^{III}, m^{II}) < i_L^{III}$. Under condition 2), if $\frac{d\Pi^A}{d\alpha_j} > 0$, we must also have $i_L^{III} \geqslant i(\alpha^*, p_L, m^{III}, m^{II})$.*

Proposition 10 states necessary and sufficient conditions to ensure a pooling ARM contract when the impact of negative house price externalities is large enough that $\alpha^* > \alpha_H^{III}$. One condition is $i^* \geqslant i(\alpha^*, p_L, m^I, m^{II})$, which guarantees that the lender has no incentive to offer a pooling FRM contract, whereas the other condition is $i(\alpha_H^{III}, p_H, m^I, m^{III}) < i(\alpha_H^{III}, p_L, m^{III}, m^{II}) < i_L^{III}$, which guarantees that the lender has no incentive to offer a separating contract. These conditions indicate that, when $\alpha^* > \alpha_H^{III}$, the value of $i_L^{III}$ needs to be high enough to ensure a pooling ARM contract. However, as we will show later, this is no longer the case when $\alpha^* \leqslant \alpha_H^{III}$.

**Proposition 11.** *Suppose $\alpha^* \leqslant \alpha_H^{III}$; then there exists a pooling equilibrium with an ARM rate $\alpha^*$ if and only if $i^* \geqslant i(\alpha^*, p_L, m^I, m^{II})$. Under condition $i_L^{III} < i(\alpha_H^{III}, p_H, m^I, m^{III}) < i(\alpha_H^{III}, p_L, m^{III}, m^{II})$ and $\frac{d\Pi^A(\alpha, p_j)}{d\alpha} < 0$, we must also have $i_L^{III} \geqslant i(\alpha^*, p_H, m^I, m^{III})$.*

Proposition 11 states necessary and sufficient conditions to ensure a pooling ARM contract when the impact of negative house price externalities is not so large that $\alpha^* \leqslant \alpha_H^{III}$. Unlike the situation in which $\alpha^* > \alpha_H^{III}$, a pooling ARM contract can exist under all ranges of $i_L^{III}$ when $\alpha^* \leqslant \alpha_H^{III}$. As a result, the likelihood of a pooling ARM contract is higher under $\alpha^* \leqslant \alpha_H^{III}$. In other words, when the impact of the negative house price externalities is moderate, a pooling ARM contract is more likely to be offered by the lender than when the impact of the negative house price externalities is large.

**Proposition 12.** *There exists a pooling equilibrium with an FRM rate $i^*$ if and only if: 1) $i^* \leqslant i(\alpha^*, p_H, m^I, m^{II})$ and $\frac{d\Pi^A(\alpha, p_j)}{d\alpha} > 0$, 2) $i^* < i(\alpha_H^{III}, p_H, m^I, m^{III})$*

An important implication of Propositions 10 and 11 is that the negative house price externalities have a negative impact on the likelihood of a pooling contract. Such impact is even more pronounced for a pooling FRM contract. As shown in Proposition 12, so long as the lender's total earnings decrease in ARM rates, a pooling FRM contract is impossible, whereas a pooling ARM contract may still exist even when the lender's total earnings decrease in ARM rates. This result is consistent with the results for a pooling ARM contract, in the sense that the negative house price externalities decrease the likelihood of a pooling contract being offered. This latter result is due to the fact that when the impact of the negative house price externalities is large, the ARM rate offered tends to be small; this makes a pooling ARM contract more attractive than a pooling FRM contract.

The rationale behind the results from Propositions 10–12 is as follows. In the presence of a negative relationship between the lender's payoff and the ARM rate, $\frac{d\Pi^A}{d\alpha_j^k} < 0$, the ARM rate offered in either a separating contract or a pooling ARM contract tends to be smaller. This has two effects on the equilibrium. One effect is on the desirability of a pooling ARM contract in relation to a separating contract. Since the lender no longer benefits from a higher ARM rate, the high-risk borrowers are more likely to be offered a lower ARM rate in a separating contract than in an ARM pooling contract. This makes a pooling ARM contract less desirable to the high-risk borrowers, and it provides less incentive for the lender to offer a pooling ARM contract than to offer a separating contract. The second effect concerns the comparison between a pooling FRM contract and a pooling ARM contract. Since the ARM rate tends to be lower, while the FRM rate is not affected by $\frac{d\Pi^A}{d\alpha_j^k} < 0$, a

[6] It is reasonable to assume $i^* > i_L^{III}$ given that $\Pi^F(i^*, p_L, m^I) > \Pi^F(i_L^{III}, p_L, m^{III}) = 0$.

[7] We are able to compare $\alpha^*$ with $\alpha_H^{II}$ because the lenders offer only ARM contracts in both situations; thus, they have the same functional form for the first-period default rate $m$.

pooling FRM contract becomes less favorable to the borrowers than a pooling ARM contract. As a result, it is more likely that the lender will prefer a pooling ARM contract to a pooling FRM contract. In summary, when $\frac{d\Pi^A}{d\alpha_j^k} < 0$, the separating contract is more likely preferred by the lender than a pooling ARM contract, and a pooling ARM contract is more likely preferred by the lender than a pooling FRM contract.

### 5.2. Separating equilibria

**Proposition 13.** *There exists a separating equilibrium* $(i_L^{III}, \alpha_H^{III})$ *with zero profits if and only if* $\alpha^* > \alpha_H^{III}$ *and* $i(\alpha_H^{III}, p_H, m^I, m^{III}) \leqslant i_L^{III} \leqslant i(\alpha_H^{III}, p_L, m^{III}, m^{II})$.

This result implies that when $\alpha^* \leqslant \alpha_H^{III}$, there does not exist a separating equilibrium with zero profits. Since $\alpha^* > \alpha_H^{III}$ more likely occurs when $\frac{d\Pi^A}{d\alpha_j^k} < 0$, it follows that the negative house price externalities increase the likelihood that a separating contract exists, which is consistent with the results from the last section. In the following proposition, we present the separating equilibria with positive profits.

**Proposition 14.** *There exists a separating equilibrium* $(\widetilde{i}_L, \alpha_H^{III})$ *where* $i(\alpha_H^{III}, p_H, m^I, m^{III}) = \widetilde{i}_L$ *if and only if: (1)* $i^* > \widetilde{i}_L$, *(2)* $\alpha^* > \alpha_H^{III}$, *and (3)* $i_L^{III} < i(\alpha_H^{III}, p_H, m^I, m^{III}) < i(\alpha_H^{III}, p_L, m^{III}, m^{II})$. *There exists a separating equilibrium* $(i_L^{III}, \widetilde{\alpha}_H)$ *where* $i(\widetilde{\alpha}_H, p_H, m^I, m^{III}) = i_L^{III}$ *if and only if: (1)* $\frac{d\Pi^A(\alpha,p_j)}{d\alpha} < 0$, *(2)* $i(\alpha^*, p_H, m^I, m^{II}) > i_L^{III}$, *and (3)* $i_L^{III} < i(\alpha_H^{III}, p_H, m^I, m^{III}) < i(\alpha_H^{III}, p_L, m^{III}, m^{II})$. *There exists a separating equilibrium* $(i_L^{III}, \widetilde{\alpha}_H)$ *where* $i_L^{III} = i(\widetilde{\alpha}_H, p_L, m^{III}, m^{II})$ *if and only if: (1)* $\frac{d\Pi^A(\alpha,p_j)}{d\alpha} > 0$, *(2)* $i(\alpha^*, p_L, m^{III}, m^{II}) > i_L^{III}$, *and (3)* $i(\alpha_H^{III}, p_H, m^I, m^{III}) < i(\alpha_H^{III}, p_L, m^{III}, m^{II}) < i_L^{III}$.[8]

Proposition 14 states three possible separating equilibria with positive profits. It shows that there may exist two separating equilibria under the condition that $i_L^{III} < i(\alpha_H^{III}, p_H, m^I, m^{III}) < i(\alpha_H^{III}, p_L, m^{III}, m^{II})$, and one separating equilibrium under the condition that $i(\alpha_H^{III}, p_H, m^I, m^{III}) < i(\alpha_H^{III}, p_L, m^{III}, m^{II}) < i_L^{III}$. Similar to the result in Proposition 13, $\alpha^* > \alpha_H^{III}$ is the necessary condition for both the first and third equilibria. Although the second equilibrium does not require $\alpha^* > \alpha_H^{III}$, the condition $\frac{d\Pi^A(\alpha,p_j)}{d\alpha} < 0$ must be satisfied. This implies that, if the impact from the negative house price externalities is not sufficiently large that $\frac{d\Pi^A(\alpha,p_j)}{d\alpha} > 0$ and $\alpha^* \leqslant \alpha_H^{III}$, a separating contract cannot exist. Once again, Proposition 14 demonstrates the crucial role that both conditions $\alpha^* > \alpha_H^{III}$ and $\frac{d\Pi^A(\alpha,p_j)}{d\alpha} < 0$ play in supporting a separating contract. This result provides insight concerning when and whether a separating contract should be adopted.

Despite the complicated nature of the model, the implication under asymmetric information is clear. As the impact from the negative house price externalities becomes larger, separating contracts become more desirable, while pooling contracts become less desirable. In the meantime, an ARM pooling contract is more likely to be offered than an FRM pooling contract. If the impact from the negative house price externalities is small enough that $\frac{d\Pi^A(\alpha,p_j)}{d\alpha} > 0$ and $\alpha^* \leqslant \alpha_H^{III}$, the lender will tend to offer pooling contracts; if the impact from the house price externalities is large enough that $\frac{d\Pi^A(\alpha,p_j)}{d\alpha} < 0$, then the lender will tend to offer separating contracts or sometimes an ARM pooling contract.

### 5.3. Comparison of default rates

In this section, we compare the default rates under asymmetric information. Let *PF* represent the pooling FRM contract, *PA* represent the pooling ARM contract, and *S* represent the separating contract. Let $(i^S, \alpha^S)$ be the rates offered in a separating contract. Then we have the following results regarding the first-period and total default rates $m^k$ and $T^k$, where $k \in \{PF, PA, S\}$.

1. $m^{PF} = \sum_{j=L,H} \lambda_j p_j$ and $T^{PF} = \sum_{j=L,H} \lambda_j p_j$
2. $m^{PA} = \sum_{j=L,H} \lambda_j \left\{ (1-p_j) \int_{Y-r-\alpha^*}^{\bar{\varepsilon}_1} f_1(\varepsilon_1) d\varepsilon_1 + p_j \int_{y-r-\alpha^*}^{\bar{\varepsilon}_1} f_1(\varepsilon_1) d\varepsilon_1 \right\}$ and $T^{PA} = \sum_{j=L,H} \lambda_j \left\{ (1-p_j) \left\{ \int_{\underline{\varepsilon}_1}^{Y-r-\alpha^*} \int_{Max\left[\underline{\varepsilon}_2, Y-r-\alpha^*-qm^{PA}\right]}^{\bar{\varepsilon}_2} f_2(\varepsilon_2) \times d\varepsilon_2 f_1(\varepsilon_1) d\varepsilon_1 + \int_{Y-r-\alpha^*}^{\bar{\varepsilon}_1} f_1(\varepsilon_1) d\varepsilon_1 \right\} + p_j \left\{ \int_{\underline{\varepsilon}_1}^{y-r-\alpha^*} \int_{Max\left[\underline{\varepsilon}_2, y-r-\alpha^*-qm^{PA}\right]}^{\bar{\varepsilon}_2} \times f_2(\varepsilon_2) d\varepsilon_2 f_1(\varepsilon_1) d\varepsilon_1 + \int_{y-r-\alpha^*}^{\bar{\varepsilon}_1} f_1(\varepsilon_1) d\varepsilon_1 \right\} \right\}$.
3. $m^S = \lambda_H \left\{ (1-p_H) \int_{Y-r-\alpha^S}^{\bar{\varepsilon}_1} f(\varepsilon_1) d\varepsilon_1 + p_H \int_{y-r-\alpha^S}^{\bar{\varepsilon}_1} f(\varepsilon_1) d\varepsilon_1 \right\} + \lambda_L p_L$ and

$$T^S = \begin{array}{l} \lambda_L p_L + \lambda_H (1-p_H) \left\{ \int_{\underline{\varepsilon}_1}^{Y-r-\alpha^S} \int_{Max\left[\underline{\varepsilon}_2, Y-r-\alpha^S-qm^S\right]}^{\bar{\varepsilon}_2} f_2(\varepsilon_2) d\varepsilon_2 f_1(\varepsilon_1) d\varepsilon_1 + \int_{Y-r-\alpha^S}^{\bar{\varepsilon}_1} f_1(\varepsilon_1) d\varepsilon_1 \right\} \\ \quad + \lambda_H p_H \left\{ \int_{\underline{\varepsilon}_1}^{y-r-\alpha^S} \int_{Max\left[\underline{\varepsilon}_2, y-r-\alpha^S-qm^S\right]}^{\bar{\varepsilon}_2} f_2(\varepsilon_2) d\varepsilon_2 f_1(\varepsilon_1) d\varepsilon_1 + \int_{y-r-\alpha^S}^{\bar{\varepsilon}_1} f_1(\varepsilon_1) d\varepsilon_1 \right\} \end{array}$$

**Proposition 15.** *If* $p_L$ *is sufficiently small,* $p_H$ *is sufficiently large, and* $\alpha^* > \alpha^S$, *then* $T^{PF}, T^{PA} > T^S$. *If both* $p_L$ *and* $p_H$ *are sufficiently small and* $\alpha^* > \alpha^S$, *then* $T^{PA} > T^S > T^{PF}$. *If both* $p_L$ *and* $p_H$ *are sufficiently large and* $\alpha^* < \alpha^S$, *then* $T^{PF} > T^S > T^{PA}$.

Proposition 15 presents sufficient conditions for ranking the default rate. It implies that if both probabilities of income shock are low and the pooling ARM rate is greater than the separating ARM rate, then the pooling FRM contract leads to the lowest default rate. If the probabilities

[8] The conditions for this equilibrium also imply that $\alpha^* > \alpha_H^{III}$.

of income shock among low-risk borrowers and high-risk borrowers differ greatly, then the separating contract leads to the lowest default rate. If the probabilities of income shock are high for both types of borrowers, then the default rate is lowest under the pooling ARM contract. The intuition for these results is straightforward: the higher the probability of an income shock, the lower the default rate will be when an ARM contract is adopted.

As mentioned earlier, $\alpha^* > \alpha^S$ indicates a higher likelihood of $\frac{d\Pi^A}{d\alpha_j^k} < 0$, and $\alpha^* < \alpha^S$ indicates a higher likelihood of $\frac{d\Pi^A}{d\alpha_j^k} > 0$. The results from the proposition above thus imply the following: (1) when the negative house price externalities have a large enough impact that $\alpha^* > \alpha^S$, the lowest default rate is likely to occur under the pooling FRM contract or the separating contract; (2) when the negative house price externalities do not have a large enough impact that $\alpha^* < \alpha^S$, the pooling ARM contract may result in the lowest default rate.

Recall from the previous section that (1) when the impact from the negative house price externalities is extremely large, a separating contract is more likely to be optimal for the lender, and (2) when the impact from the negative house price externalities is small enough that the lender's profit is still increasing in the ARM rate, pooling contracts are more likely to occur.[9] Although the results from the default rate comparison are not completely in line with those for the optimal contracts, they do overlap to a large extent. For a social planner or a government agency that aims to control the default rate under large house price externalities, therefore, special attention should be paid to the situation in which the pooling FRM contract offers the lowest default rate while $\alpha^* > \alpha^S$. Proposition 15 states that if both $p_L$ and $p_H$ are sufficiently small and $\alpha^* > \alpha^S$, then the pooling FRM contract offers the lowest default rate. According to the results from the previous section, when $\alpha^* > \alpha^S$, the optimal contract for the lender is likely to be a separating contract in this situation, and this may lead to a higher default rate. The rationale for an equilibrium contract with a higher default rate lies in two driving forces: the low risk of an income shock drives the low default rate under a pooling FRM contract, while the presence of large house price externalities makes the separating contract optimal for the lender. Since both forces operate in this particular situation, an equilibrium contract may not be socially desirable if the goal is to minimize the default rate.

## 6. Conclusion

In this paper, we have considered the relationships among mortgage product choice, negative house price externalities, and the default rate. The results suggest that the presence of house price externalities has important implications for the equilibrium menu of mortgage contracts offered by lenders and adopted by borrowers, as well as for the overall default rate. In the presence of negative house price externalities, a higher ARM rate triggers an additional loss of earnings that would not otherwise occur to the lender. As a result, the lender's profits may be decreasing in the ARM rate, and thus the equilibrium ARM rate in a pooling equilibrium may exceed the equilibrium ARM rate that would be offered to high-risk borrowers in a separating equilibrium. In consequence, when externalities are large, the lender prefers to offer separating contracts, and high-risk borrowers receive a lower interest rate than they would have received under a pooling contract. When house price externalities are small, the equilibrium will tend toward pooling contracts. The ranking of the default rates follows a similar pattern as the mortgage contracts. We have shown that when the negative house price externalities have a large enough impact, the pooling FRM contract or the separating contract tends to offer the lowest default rate; when the negative house price externalities do not have a large enough impact, the pooling ARM contract may result in the lowest default rate.

## Appendix A

**Proof** (*Proof of Lemma 1*). Taking the derivative $\Delta V(i,\alpha;p_j)$ with respect to $p_j$ yields

$$\begin{aligned}\frac{\partial \Delta V(i,\alpha;p_j)}{\partial p_j} = & -U(Y-i)+U(0)-D-\delta U(Y-i-qm)\\ & +\left\{\int_{\underline{\varepsilon}_1}^{Y-r-\alpha}(U(Y-r-\alpha-\varepsilon_1)+\delta\widehat{A})f_1(\varepsilon_1)d\varepsilon_1+\int_{Y-r-\alpha}^{\bar{\varepsilon}_1}(U(0)-D)f_1(\varepsilon_1)d\varepsilon_1\right\}\\ & -\left\{\int_{\underline{\varepsilon}_1}^{y-r-\alpha}(U(y-r-\alpha-\varepsilon_1)+\delta\widehat{B})f_1(\varepsilon_1)d\varepsilon_1+\int_{y-r-\alpha}^{\bar{\varepsilon}_1}(U(0)-D)f_1(\varepsilon_1)d\varepsilon_1\right\}.\end{aligned}$$

One can see easily that when $D$ is sufficiently large, $\frac{\partial \Delta V(i,\alpha;p_j)}{\partial p_j} < 0$. □

**Proof** (*Proof of Proposition 2*). By the implicit function theorem, we have

$$\partial i(\alpha,p,j)/\partial p_j = -\frac{\partial \Delta V(i,\alpha;p_j)/\partial p_j}{\partial \Delta V(i,\alpha;p_j)/\partial i}.$$

Since $\partial \Delta V(i,\alpha;p_j)/\partial p_j < 0$ and $\partial \Delta V(i,\alpha;p_j)/\partial i < 0$, it follows that $\partial i(\alpha,p,j)/\partial p_j < 0$. □

**Lemma 16.** *If* $\frac{d\Pi^A}{d\alpha_j^k} > 0$, *then* $(m^{II}-m^{III})(\alpha_H^{II}-\alpha_H^{III}) > 0$; *if* $\frac{d\Pi^A}{d\alpha_j^i} < 0, (m^{II}-m^{III})(\alpha_H^{II}-\alpha_H^{III}) < 0$. *If* $p_H$ *is sufficiently large, then* $m^I > m^{III}$.

**Proof.** By Lemma 4, $\frac{\partial \Pi^A}{\partial m} < 0$. When $\frac{d\Pi^A}{d\alpha_j^k} > 0, m^k$ and $\alpha_j^k$ affect total earnings in opposite directions. Under such circumstance, in order to satisfy the zero-profit contract condition, a higher value of $m$ must accompany a higher value of $\alpha_j^i$. Therefore, the case with a higher ARM rate will have a higher first-period default rate, and vice-versa. When $\frac{d\Pi^A}{d\alpha_j^k} < 0, m$ and $\alpha_j^k$ affect total earnings in the same direction. Thus, a higher (lower) value of $m$ must accompany a lower (higher) value of $\alpha_j^k$. As a result, the case with a higher ARM rate will have a lower first-period default rate, and vice-versa. □

[9] This causes no contradiction with the results of Posey and Yavas (2001) since both pooling and separating contracts are possible in their paper.

**Proof** (*Proof of Proposition 6*). By Lemma 16, when $\frac{d\Pi^A}{d\alpha_j^k} < 0$, a higher (lower) value of $m$ must accompany a lower (higher) value of $\alpha_j^k$. Thus, we have either $\alpha_H^{II} < \alpha_H^{III}$ and $m^{II} > m^{III}$, or $\alpha_H^{II} > \alpha_H^{III}$ and $m^{II} < m^{III}$ . When $p_L$ is sufficiently small, we can see easily that low-risk borrowers have a higher default rate in Case II than in Case III. For high-risk borrowers, the relative default rate depends on the value of $\alpha_H^k$, since the default rate function is the same for both cases. Also, we know that $\frac{dm^k}{d\alpha_j^k} > 0$, so the default rate for high-risk borrowers is increasing in $\alpha_H$. Suppose that $\alpha_H^{II} > \alpha_H^{III}$; then the default rate for high-risk borrowers is higher in Case II than in Case III. Since low-risk borrowers also have a higher default rate in Case II than in Case III, it must be true that $m^{II} > m^{III}$, which causes contradiction. Therefore, if $\frac{d\Pi^A}{d\alpha_j^k} < 0$ and $p_L$ is sufficiently small, then $\alpha_H^{II} < \alpha_H^{III}$. When $p_L$ is sufficiently large, the proof follows the same steps, so we omit it for simplicity. □

**Proof** (*Proof of Proposition 10*). The conditions in part (1) guarantee that the lender has no incentive to offer a pooling FRM contract, while the conditions in part (2) guarantee that the lender has no incentive to deviate to a separating contract. To prove part (2), it is necessary to have $i_L^{III} < i(\alpha_H^{III}, p_H, m^I, m^{III}) < i(\alpha_H^{III}, p_L, m^{III}, m^{II})$ or $i(\alpha_H^{III}, p_H, m^I, m^{III}) < i(\alpha_H^{III}, p_L, m^{III}, m^{II}) < i_L^{III}$, since otherwise the lender has an incentive to deviate to a separating contract $(i_L^{III}, \alpha_H^{III})$ due to the fact that $\alpha^* > \alpha_H^{III}$. First, consider the situation in which $i_L^{III} < i(\alpha_H^{III}, p_H, m^I, m^{III}) < i(\alpha_H^{III}, p_L, m^{III}, m^{II})$. In this situation, there exists a separating contract $(\tilde{i}_L, \alpha_H^{III})$ where $\tilde{i}_L = i(\alpha_H^{III}, p_H, m^I, m^{III})$. Since this contract is more attractive to the lender, this situation cannot occur. Next, consider the situation in which $i(\alpha_H^{III}, p_H, m^I, m^{III}) < i(\alpha_H^{III}, p_L, m^{III}, m^{II}) < i_L^{III}$. In this situation, there exists a separating contract $(i_L^{III}, \tilde{\alpha}_H)$ where $\tilde{\alpha}_H > \alpha_H^{III}$ if $\frac{d\Pi^A(\alpha, p_j)}{d\alpha} > 0$. Unless $\alpha^* \leqslant \tilde{\alpha}_H$, this contract would be more attractive to the lender as well. As a result, we must have the condition $\alpha^* \leqslant \tilde{\alpha}_H$, which implies $i_L^{III} \geqslant i(\alpha^*, p_L, m^{III}, m^{II})$. □

**Proof** (*Proof of Proposition 11*). Since $\alpha^* \leqslant \alpha_H^{III}$, the only possible attractive separating contract is $(i_L^{III}, \tilde{\alpha}_H)$ when $i_L^{III} < i(\alpha_H^{III}, p_H, m^I, m^{III}) < i(\alpha_H^{III}, p_L, m^{III}, m^{II})$. According to case II, in this situation it is necessary to have $\alpha^* \leqslant \tilde{\alpha}_H$, which requires $i(\alpha^*, p_H, m^I, m^{III}) \leqslant i_L^{III}$ if $\frac{d\Pi^A(\alpha, p_j)}{d\alpha} < 0$. □

**Proof** (*Proof of Proposition 12*). Suppose that there exists a pooling FRM contract; then, the lender must have no incentive to deviate to a pooling ARM contract. This implies that

$$\begin{cases} \Delta V(i^*, \alpha, p_L) = V^F(i^*, p_L, m^I) - V^A(\alpha, p_L, m^{II}(\alpha)) \geqslant 0 \\ \Delta V(i^*, \alpha, p_H) = V^F(i^*, p_H, m^I) - V^A(\alpha, p_H, m^{II}(\alpha)) \geqslant 0 \end{cases} \tag{7}$$

for all $\alpha$ such that

$$\lambda \Pi^A(\alpha, p_L, m^{II}) + (1-\lambda)\Pi^A(\alpha, p_H, m^{II}) \geqslant 0 \tag{8}$$

The conditions (7) imply that $i^* \leqslant i(\alpha, p_H, m^I, m^{II}) < i(\alpha, p_L, m^I, m^{II})$. Suppose that $\frac{d\Pi^A(\alpha, p_j)}{d\alpha} > 0$; then the condition (8) implies that $\alpha \geqslant \alpha^*$. As a result, if $i^* \leqslant i(\alpha^*, p_H, m^I, m^{II})$, the lender has no incentive to deviate to a pooling ARM contract. Thus, $i^* \leqslant i(\alpha^*, p_H)$ is necessary and sufficient to exclude an ARM contract. Suppose that $\frac{d\Pi^A(\alpha, p_j)}{d\alpha} < 0$; then the condition (8) implies that $\alpha \leqslant \alpha^*$. As a result, the condition $i^* \leqslant i(\alpha^*, p_H, m^I, m^{II})$ does not guarantee no deviation to a pooling ARM contract unless $\alpha = \alpha^*$. Thus, when $\frac{d\Pi^A(\alpha, p_j)}{d\alpha} < 0$, the condition $i^* \leqslant i(\alpha^*, p_H)$ is necessary but insufficient to exclude an ARM contract.

The conditions in part (1) guarantee that the lender has no incentive to offer a pooling ARM contract, while the conditions in part (2) guarantees that the lender has no incentive to deviate to a separating contract. To prove part 2), it is necessary to have $i_L^{III} < i(\alpha_H^{III}, p_H, m^I, m^{III}) < i(\alpha_H^{III}, p_L, m^{III}, m^{II})$ or $i(\alpha_H^{III}, p_H, m^I, m^{III}) < i(\alpha_H^{III}, p_L, m^{III}, m^{II}) < i_L^{III}$ since, otherwise the lender has an incentive to deviate to a separating contract $(i_L^{III}, \alpha_H^{III})$. Since $i^* > i_L^{III}$, the separating contract $(i_L^{III}, \alpha_H^{III})$ will attract both borrowers away for a pooling FRM contract.with $i^*$. First, consider the situation in which $i_L^{III} < i(\alpha_H^{III}, p_H, m^I, m^{III}) < i(\alpha_H^{III}, p_L, m^{III}, m^{II})$. In this situation, there exists a separating contract $(\tilde{i}_L, \alpha_H^{III})$ where $\tilde{i}_L = i(\alpha_H^{III}, p_H, m^I, m^{III})$. Unless $i^* < \tilde{i}_L$, this separating contract will be more attractive than the pooling FRM contract. Since $\frac{d\Pi^A(\alpha, p_j)}{d\alpha} > 0$ , the separating contract $(i_L^{III}, \tilde{\alpha}_H)$ where $i(\tilde{\alpha}_H, p_H, m^I, m^{III}) = i_L^{III}$ does not exist. Next, consider the situation when $i(\alpha_H^{III}, p_H, m^I, m^{III}) < i(\alpha_H^{III}, p_L, m^{III}, m^{II}) < i_L^{III}$. In this situation, there exists a separating contract $(i_L^{III}, \tilde{\alpha}_H)$. Since $i^* > i_L^{III}$, this contract would be more attactive, thus this situation must be ruled out. As a result, we must have $i_L^{III} < i(\alpha_H^{III}, p_H, m^I, m^{III}) < i(\alpha_H^{III}, p_L, m^{III}, m^{II})$ and $i^* < \tilde{i}_L$ which implies $i^* < i(\alpha_H^{III}, p_H, m^I, m^{III})$. Part 2) is proved. □

**Proof** (*Proof of Proposition 13*). According to the discussion in case I under asymmetric information, when $i(\alpha_H^{III}, p_H, m^I, m^{III}) \leqslant i_L^{III} \leqslant i(\alpha_H^{III}, p_L, m^{III}, m^{II})$, there exists a separating contract $(i_L^{III}, \alpha_H^{III})$ as long as there does not exist a pooling contract. Next, we will prove that no pooling contract exists that will attract both borrower types. Since $i^* > i_L^{III}$ and $\alpha^* > \alpha_H^{III}$, a pooling FRM contract with $i^*$ and a pooling ARM contract with $\alpha^*$ can never attract both borrowers types. □

**Proof** (*Proof of Proposition 14*). The proof of Proposition 14 follows the same logic as the proof of Proposition 13. To ensure the existence of a separating equilibrium, the rates must be offered so that no pooling contract can attract both borrower types. That is why $i^* > \tilde{i}_L$ and $\alpha^* > \alpha_H^{III}$ are necessary for the first separating equilibrium. For the second separating equilibrium, the condition $i(\alpha^*, p_H, m^I, m^{II}) > i_L^{III}$ implies $\alpha^* > \tilde{\alpha}_H$, thus it is necessary. For the third separating equilibrium, the condition $i(\alpha^*, p_L, m^{III}, m^{II}) > i_L^{III}$ implies $\alpha^* > \widetilde{\alpha}_H$, thus it is necessary as well. □

**Proof** (*Proof of Proposition 15*). If $p_H$ is sufficiently large, then $T^{PF} > T^S$, if $p_H$ is sufficiently small, then $T^{PF} < T^S$. If $p_L$ is sufficiently small and $\alpha^* > \alpha^S$, then $T^{PA} > T^S$; if $p_L$ is sufficiently large and $\alpha^* < \alpha^S$, then $T^{PA} < T^S$. The rest of the rankings follows. □

**Proposition 17.** *In summary, we have the following cases for potential separating equilibria:*

1. When $i(\alpha_H^{III}, p_H, m^I, m^{III}) \leqslant i_L^{III} \leqslant i(\alpha_H^{III}, p_L, m^{III}, m^{II})$, there may exist a separating contract $(i_L^{III}, \alpha_H^{III})$ with zero profits;
2. When $i_L^{III} < i(\alpha_H^{III}, p_H, m^I, m^{III}) < i(\alpha_H^{III}, p_L, m^{III}, m^{II})$,
   - there may exist a separating contract $(\widetilde{i}_L, \alpha_H^{III})$ where $i(\alpha_H^{III}, p_H, m^I, m^{III}) = \widetilde{i}_L < i(\alpha_H^{III}, p_L, m^{III}, m^{II})$ and $\widetilde{i}_L > i_L^{III}$;
   - if $\frac{d\Pi^A(\alpha, p_j)}{d\alpha} < 0$, there may exist another separating contract $(i_L^{III}, \widetilde{\alpha}_H)$ where $i(\widetilde{\alpha}_H, p_H, m^I, m^{III}) = i_L^{III} < i(\widetilde{\alpha}_H, p_L, m^{III}, m^{II})$ and $\widetilde{\alpha}_H < \alpha_H^{III}$.
3. When $i(\alpha_H^{III}, p_H, m^I, m^{III}) < i(\alpha_H^{III}, p_L, m^{III}, m^{II}) < i_L^{III}$,
   - if $\frac{d\Pi^A(\alpha, p_j)}{d\alpha} > 0$, there may exist a separating contract $(i_L^{III}, \widetilde{\alpha}_H)$ where $i(\widetilde{\alpha}_H, p_H, m^I, m^{III}) < i_L^{III} = i(\widetilde{\alpha}_H, p_L, m^{III}, m^{II})$ and $\widetilde{\alpha}_H > \alpha_H^{III}$.
   - if $\frac{d\Pi^A(\alpha, p_j)}{d\alpha} < 0$, there does not exist a separating contract.

**Proof** (*Proof of Proposition 17*). Under a separating equilibrium, low-risk borrowers will choose the FRM contract and high-risk borrowers will choose the ARM contract. Consider a zero profit contract $(i_L^{III}, \alpha_H^{III})$. For type L borrowers, the incentive compatibility constraint is

$$\Delta V(i_L^{III}, \alpha_H^{III}, p_L) = V^F(i_L^{III}, p_L, m^{III}) - V^A(\alpha_H^{III}, p_L, m^{II}(\alpha_H^{III})) \geqslant 0. \tag{9}$$

Let $i(\alpha_H^{III}, p_L, m^{III}, m^{II})$ be the FRM rate such that $\Delta V(i, \alpha_H^{III}, p_L) = 0$. Since $\frac{\partial \Delta V(i, \alpha; p_j)}{\partial i} < 0$, it follows that $i_L^{III} \leqslant i(\alpha_H^{III}, p_L, m^{III}, m^{II})$. For type H borrowers, the incentive compatibility constraint is $\Delta V(i_L^{III}, \alpha_H^{III}, p_H) = V^F(i_L^{III}, p_H, m^I) - V^A(\alpha_H^{III}, p_H, m^{III}) \leqslant 0$. Let $i(\alpha_H^{III}, p_H, m^I, m^{III})$ be the FRM rate such that $\Delta V(i, \alpha_H^{III}, p_H) = 0$. It follows that $i_L^{III} \geqslant i(\alpha_H^{III}, p_H, m^I, m^{III})$. Thus, we have the following necessary conditions for zero-profit separating contracts.

$$i(\alpha_H^{III}, p_H, m^I, m^{III}) \leqslant i_L^{III} \leqslant i(\alpha_H^{III}, p_L, m^{III}, m^{II}).$$

The separating equilibria with positive profits exist either $i_L^{III} < i(\alpha_H^{III}, p_H, m^I, m^{III}) < i(\alpha_H^{III}, p_L, m^{III}, m^{II})$ or $i(\alpha_H^{III}, p_H, m^I, m^{III}) < i(\alpha_H^{III}, p_L, m^{III}, m^{II}) < i_L^{III}$. Suppose $i_L^{III} < i(\alpha_H^{III}, p_H, m^I, m^{III}) < i(\alpha_H^{III}, p_L, m^{III}, m^{II})$; then it will not be incentive compatible for type H borrowers to choose $\alpha_H^{III}$ if $i_L^{III}$ is offered. In order to satisfy incentive compatibility for both types of borrowers, the lender must increase the FRM rate or decrease the ARM rate. Suppose that the lender increases the FRM rate. Let $\widetilde{i}_L$ be the FRM rate such that $i(\alpha_H^{III}, p_H, m^I, m^{III}) \leqslant \widetilde{i}_L \leqslant i(\alpha_H^{III}, p_L, m^{III}, m^{II})$. Consequently, the lender makes positive profits from type L borrowers. In a competitive market, $\widetilde{i}_L = i(\alpha_H^{III}, p_H, m^I, m^{III})$. Now consider the option of decreasing the ARM rate. Let $\widetilde{\alpha}_H$ be the ARM rate such that $i(\widetilde{\alpha}_H, p_H, m^I, m^{III}) \leqslant i_L^{III} \leqslant i(\alpha_H^{III}, p_L, m^{III}, m^{II})$. Suppose that $\frac{d\Pi^A(\alpha, p_j)}{d\alpha} > 0$; then decreasing $\alpha_H^{III}$ will result in negative payoffs, which will not be adopted. Suppose $\frac{d\Pi^A(\alpha, p_j)}{d\alpha} < 0$; then decreasing $\alpha_H^{III}$ will result in positive payoffs. In a competitive market, the ARM rate $\widetilde{\alpha}_H$ must satisfy $i(\widetilde{\alpha}_H, p_H, m^I, m^{III}) = i_L^{III}$.Suppose that $i(\alpha_H^{III}, p_H, m^I, m^{III}) < i(\alpha_H^{III}, p_L, m^{III}, m^{II}) < i_L^{III}$; then condition (9) indicates that it would not be incentive compatible for type L borrowers to choose $i_L^{III}$ if $\alpha_H^{III}$ is offered. In order to satisfy incentive compatibility for both types of borrowers, the lender must decrease the FRM rate or increase the ARM rate. Decreasing the FRM rate will result in a negative payoff, which will not be adopted. Increasing the ARM rate is possible only if $\frac{d\Pi^A(\alpha, p_j)}{d\alpha} > 0$, since it would result in negative payoffs otherwise. In a competitive market, the ARM rate $\widetilde{\alpha}_H$ must satisfy $i(\widetilde{\alpha}_H, p_L, m^{III}, m^{II}) = i_L^{III}$. □

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